REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* 08-08-2013	**2. REPORT TYPE** Final Technical	**3. DATES COVERED** *(From - To)* May 2010 - April 2013

4. TITLE AND SUBTITLE
Multi-scale Uncertainty Propagation in Dynamical Systems

5a. CONTRACT NUMBER

5b. GRANT NUMBER
FA9550-10-1-0143

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Bamieh, Bassam, UCSB
Mezic, Igor, UCSB
Khammash, Mustafa, UCSB
Rowley, Clarence, W., Princeton

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The Regents of the University of California
Office of Research, 3227 Cheadle Hall
Santa Barbara, CA 93106-2050
Princeton University, Princeton, NJ 08544

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Air Force Office of Scientific Research
875 N. Randolph St., Rm 3112
Arlington, VA 22203-1954

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Theoretical and computational methods to analyze and control the dynamic behavior of complex systems under uncertainty were investigated. Compressive Polynomial Chaos Expansions were used to circumvent the large-scale difficulties common in other Polynomial Chaos expansions. In the area of Koopman and Dynamic Mode Decomposition Analysis, stable and efficient computational techniques were developed that address a suite of problems, from Ergodic Quotient computations to complex turbulent flow characterizations. This resulted in a Koopman mode theory that rigorously unifies a number of seemingly distinct concepts advanced in fluid dynamics. Using the setting of stochastic structured uncertainty, a purely input-output theory of systems with time-varying stochastic parameters was developed. New mean-square stability tests were discovered with two important features, computational complexity that scales with number of uncertainties rather than with state dimension, and the ability to handle correlated uncertainty. Distributed control design in large-scale stochastic networks was studied. In the limit of large system size, surprising dimensionality dependencies and phase transition phenomena were discovered in the optimal control design problem itself.

15. SUBJECT TERMS
Uncertainty Propagation, Uncertainty Quantification, Distributed Control, Robust Control, Polynomial Chaos, Complex Systems

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** Dr. Bassam Bamieh
a. REPORT U	**b. ABSTRACT** U	**c. THIS PAGE** U	UU		**19b. TELEPHONE NUMBER** *(include area code)* (805) 893-4490

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39.18

Multi-scale Uncertainty Propagation in Dynamical Systems

Final Report
FA9550-10-1-0143

Bassam Bamieh, Mustafa Khammash and Igor Mezic

Department of Mechanical Engineering

University of California at Santa Barbara

Clarence Rowley

Mechanical & Aerospace Engineering

Princeton University

August 8, 2013

Abstract

Theoretical and computational methods to analyze and control the dynamic behavior of complex systems under uncertainty were investigated. Compressive Polynomial Chaos Expansions were used to circumvent the large-scale difficulties common in other Polynomial Chaos expansions. In the area of Koopman and Dynamic Mode Decomposition Analysis, stable and efficient computational techniques were developed that address a suite of problems, from Ergodic Quotient computations to complex turbulent flow characterizations. This resulted in a Koopman mode theory that rigorously unifies a number of seemingly distinct concepts advanced in fluid dynamics. Using the setting of stochastic structured uncertainty, a purely input-output theory of systems with time-varying stochastic parameters was developed. New mean-square stability tests were discovered with two important features, computational complexity that scales with number of uncertainties rather than with state dimension, and the ability to handle correlated uncertainty. Distributed control design in large-scale stochastic networks was studied. In the limit of large system size, surprising dimensionality dependencies and phase transition phenomena were discovered in the optimal control design problem itself.

1 Summary

The original aim of this project was the development of a framework that accelerates the computational investigation and control of complex physical models under uncertainty. The underlying principle was not the development of a single methodology that is equally applicable to any uncertain dynamical system. Such overarching methodologies, while general in applicability, can always be shown to have deficiencies in certain examples. The underlying principle was rather to build from the ground up, by using specific methods on certain classes of problems, and develop abstractions and generalizations when appropriate. This bottoms-up approach appears to be more natural and effective in developing powerful methodologies that are applicable to problems of interest to the scientific and engineering communities. Significant progress has been made in four thrust areas. These are Compressive Polynomial Chaos (CPC) Expansions, Koopman/Dynamic Mode Decomposition (DMD) techniques, dynamical Structured Stochastic Uncertainty, and phase transitions in large-scale stochastic networks. A common feature of these results are new algorithms suitable for large-scale problems.

Compressive Polynomial Chaos is a technique developed in the course of this project [1]. It is a novel computationally tractable technique for computing the coefficients of polynomial chaos expansions based on ℓ^1 and convex optimization. The approach can be applied to problems with a large number of random variables and uses a modest number of Monte Carlo simulations while avoiding model manipulations. This technique has also been applied to stochastic model predictive control [2] where the computationally expensive step of cost function evaluation was performed using the CPC expansion.

In the area of Koopman/DMD analysis, a theory was developed that describes a sense in which DMD approximates the action of the Koopman operator on an appropriate Krylov subspace. It was shown that Koopman mode theory unifies and provides a rigorous background for a number of different concepts that have been advanced in fluid mechanics [3, 4, 5], including Global Mode Analysis, triple decomposition and Dynamic Mode Decomposition. In addition, important relationships have been discovered between Koopman, DMD and Fourier analyses. Using only data as snapshots of a vector-valued observable, it was shown that DMD algorithms have several advantages over the Discrete Fourier Transform (DFT). Variants of the Balanced POD algorithm have been developed that in one instance dramatically reduce the required number of snapshots of the state, and in another uses the restriction of the state to the controllable subspace which often has much smaller dimension in control problems with limited actuation [6, 7, 8, 9, 10, 11].

In dynamical systems with structured stochastic uncertainty new necessary and sufficient conditions for mean square stability have been obtained that are tractable for large-scale systems. These conditions were obtained through the development of a purely input-output theory of dynamic systems with structured stochastic perturbations [12]. The loss of mean square stability has recently been shown to indicate the emergence of complex behavior in networked dynamical systems, and these conditions are thus very relevant for apriori predictions (i.e. pre-simulations) of the emergence of such behavior. These results include necessary

and sufficient conditions for the physically important case of correlated uncertainties, which has previously been unresolved.

Distributed control in stochastic networks is an area of intense current research. The central question of performance limitations due to network structure was studied in the both the general setting as well as the setting of vehicular formations and consensus-like dynamical networks. For large-scale networks, asymptotic results that quantify the limits of achievable performance were obtained [13, 14, 15]. In particular, a clear answer to the question of why the so-called vehicular platoons problem is inherently difficult for large systems was obtained, in that it is spatially a one dimensional problem. These results made strong contact with the problems of harmonic solids and order-disorder transitions in statistical mechanics. This result is one of few in the distributed control literature that directly connect limits of performance statements with network topology, a significant question in this field.

2 Personnel Supported

Aside from the four co-PIs, the following postdoctoral and graduate researchers worked on, and were supported or partially supported by this project.

Postdoctoral Researchers

- Bryan Eisenhower (UCSB)
- Alice Hubenko (UCSB)
- Paul Kauffman (UCSB)
- Dirk Martin (Princeton)
- Stacy Patterson (UCSB)

Graduate Student Researchers

- Carla Bahri (Princeton)
- Marko Budisic (UCSB)
- Mitchel Craun (UCSB)
- Anthony DeGennaro (Princeton)
- Jonathan Epperlein (UCSB)
- Ryan Mohr (UCSB)
- Gunjan Thakur (UCSB)

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